PHYS 231 – Oct. 3, 2023

- Pls. submit Assign. #1 on Thursday, Oct. 5, 2023 at the start of your lab.

- Assignment #2 will be posted on the course website this week.

Previously: Solved differential equations to find $i(t)$ in RC & LRC circuits
→ Transient response.

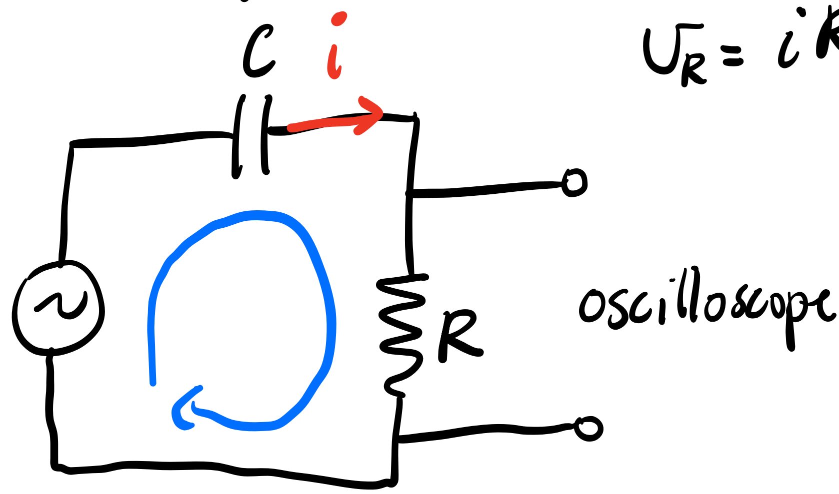
→ You will analyze the transient response of an LR circuit in assignment #2.

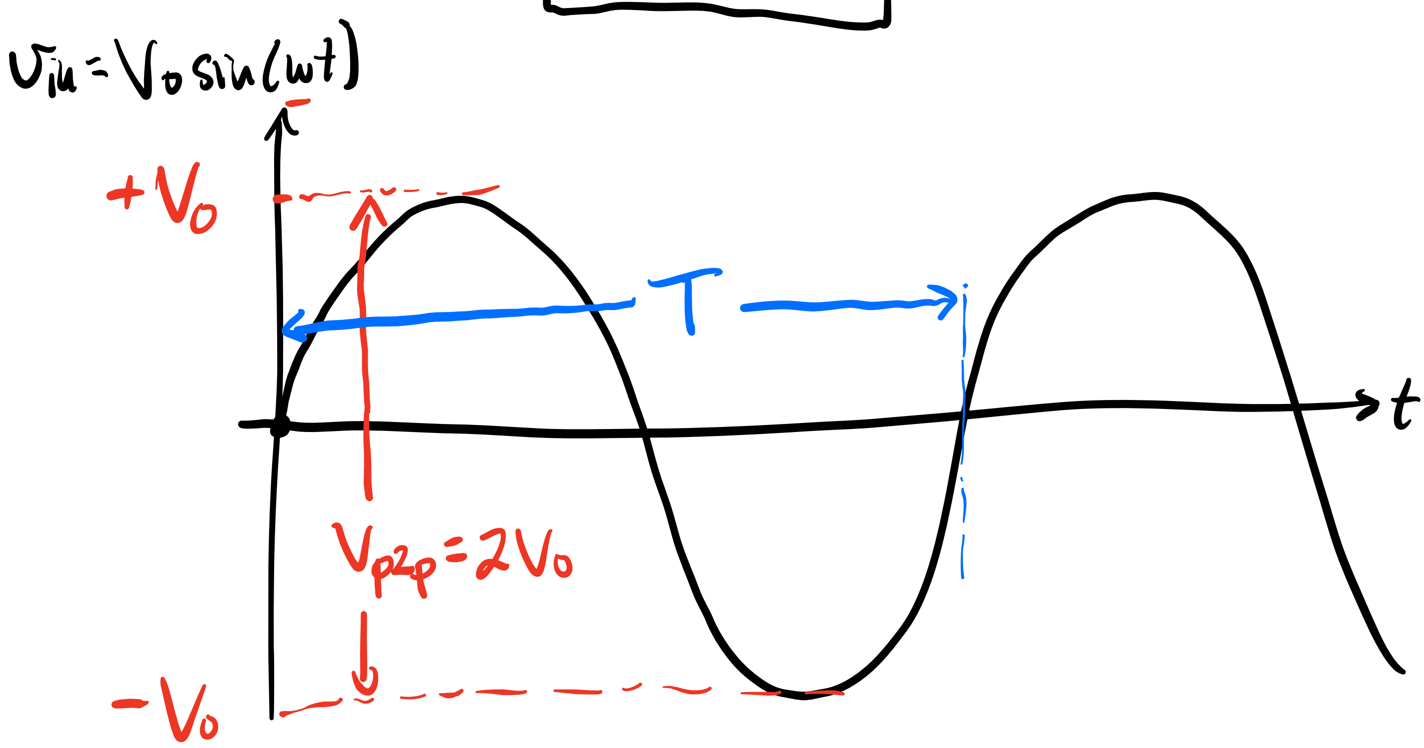
→ Today: Analyze the frequency response of an RC circuit. i.e. how does i vary with frequency (instead of time).

RC Circuit Freq Responce

$v_R = iR$

$v_{in} = V_0 \sin(\omega t)$





V_0 : amplitude

T : period (time to complete one full cycle).

$f = \frac{1}{T}$ is the frequency. $[f] = Hz$

$\omega = 2\pi f = \frac{2\pi}{T}$: angular freq.

$[\omega] = \frac{rad}{s}$

Observations from Part 4 of Lab #2:

- current also osc. sinusoidally & has the same freq. as v_{in}.

- current is out of phase w/ v_{in}. The obs. phase depends on ω.

- The amplitude of the current depends on ω.

Assume that the current in the RC circuit is of the form:

$$i = \underbrace{I_0(\omega)}_{\text{freq. depend. amplitude}} \sin\Big[\overset{\text{same freq. as } v_{in}}{\omega} t + \underbrace{\phi(\omega)}_{\text{freq. depend. phase.}}\Big]$$

Goal: Determine how $I_0(\omega)$ & $\phi(\omega)$ depend on freq. & the circuit components.

KVLR:

$$v_{in} - V_C - V_R = 0$$

$$v_{in} - \frac{q}{C} - iR = 0$$

Take a time derivative of this expression

$$\text{(\#)} \quad \frac{dv_{in}}{dt} = \frac{1}{C} i + R\frac{di}{dt}$$

differential eq'n in i.

$$\left.\begin{aligned} &\text{Know } v_{in} = V_0 \sin \omega t \\ &\frac{dv_{in}}{dt} = \omega V_0 \cos \omega t \\ &i = I_0 \sin(\omega t + \phi) \\ &\frac{di}{dt} = \omega I_0 \cos(\omega t + \phi) \end{aligned}\right\} \text{sub into (\#)}$$

$$\omega V_0 \cos \omega t = \frac{I_0}{C} \sin(\omega t + \phi) + \omega I_0 R \cos(\omega t + \phi)$$

Use the following trig identities:

$$\sin(\omega t + \phi) = \sin \omega t \cos \phi + \cos \omega t \sin \phi$$

$$\cos(\omega t + \phi) = \cos \omega t \cos \phi - \sin \omega t \sin \phi$$

$$\omega V_0 \underline{\cos \omega t} = \frac{I_0}{C}\left(\underline{\sin \omega t} \cos \phi + \underline{\cos \omega t} \sin \phi\right)$$

$$+ \omega I_0 R \left(\underline{\cos \omega t} \cos \phi - \underline{\sin \omega t} \sin \phi\right)$$

$$\left\{ \begin{aligned} &\sin \omega t \left(\frac{I_0}{C} \cos \phi - \omega I_0 R \sin \phi\right) \\ &+ \cos \omega t \left(\frac{I_0}{C} \sin \phi + \omega I_0 R \cos \phi - \omega V_0\right) \end{aligned} \right.$$

$$= 0$$

This expression must be valid at ALL times.

at some particular times will have

$\sin \omega t = 0$ & $\cos \omega t \neq 0$.

At these times, require

① $$\boxed{\frac{I_0}{C}\underbrace{\sin\phi}_{a} + \omega I_0 R \underbrace{\cos\phi}_{b} - \omega V_0 = 0}$$

At other times, $\cos \omega t = 0$ & $\sin \omega t \neq 0$

$\therefore$ it must also be true that:

② $$\boxed{\frac{\cancelto{1}{I_0}}{C}\cos\phi - \omega \cancelto{1}{I_0} R \sin\phi = 0}$$

Now, we try to solve Eq'ns ① & ② for $\phi(\omega)$ & $I_0(\omega)$.

Start w/ Eq'n 2.

$$\frac{1}{C}\cos\phi = \omega R\sin\phi$$

$\Downarrow$ divide by $\omega R\cos\phi$

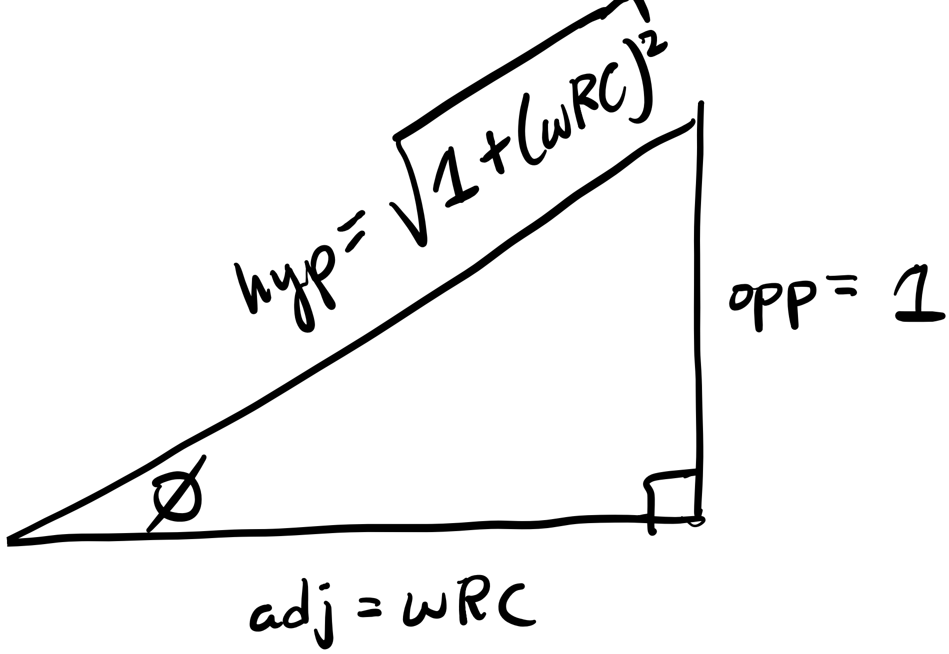
$$\frac{1}{\omega RC} = \tan\phi \Rightarrow \boxed{\phi(\omega) = \tan^{-1}\left(\frac{1}{\omega RC}\right)}$$

Freq. dependence of the phase of the current.

Return to

$$\tan\phi = \frac{\sin\phi}{\cos\phi} = \frac{\text{opp}}{\text{adj}} = \frac{1}{\omega RC}$$

Strategy: Construct a right-angle triangle w/ opp = 1 adj = ωRC



$$\sin\phi = \frac{opp}{hyp} = \frac{1}{\sqrt{1+(\omega RC)^2}} \qquad (a)$$

$$\cos\phi = \frac{adj}{hyp} = \frac{\omega RC}{\sqrt{1+(\omega RC)^2}} \qquad (b)$$

Sub (a) & (b) into Eq'n (1).

Eq'n (1): $\frac{I_0}{C}\sin\phi + \omega I_0 R\cos\phi - \omega V_0 = 0$

$$\rightarrow \frac{I_0}{C}\frac{1}{\sqrt{1+(\omega RC)^2}} + \omega I_0 R\frac{\omega RC}{\sqrt{1+(\omega RC)^2}} - \omega V_0 = 0$$

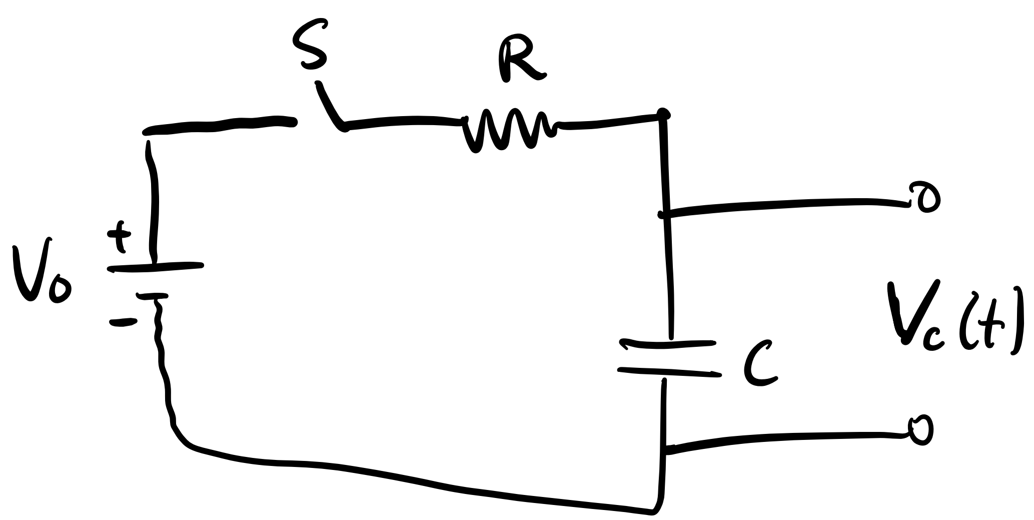
solve for $I_0(\omega)$ $\rightarrow$ Exercise for the student

$$\boxed{I_0 = \frac{\omega V_0 C}{\sqrt{1+(\omega RC)^2}}}$$

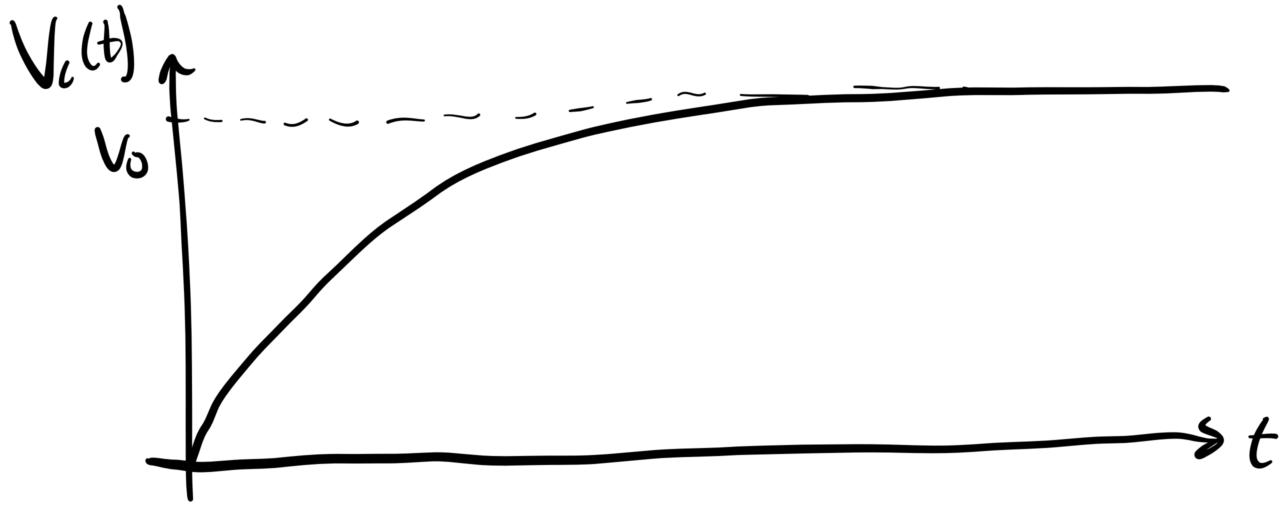
How the current amplitude I_0 varies w/ freq. ω.

Lab #3 Preview

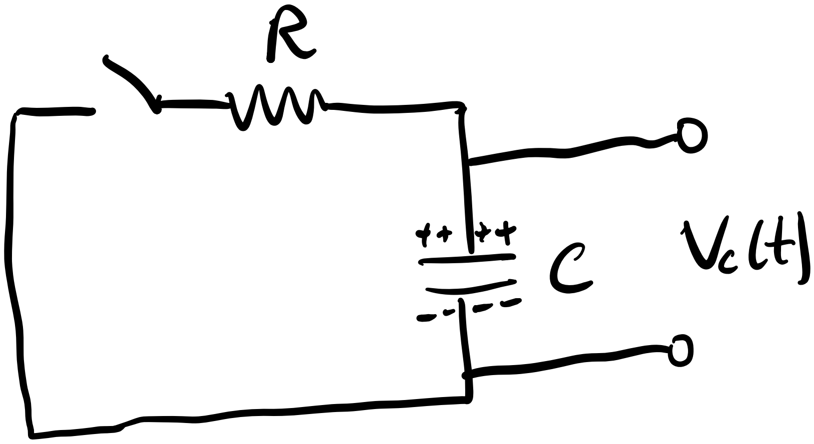
Charging a capacitor through a resistor



$$V_c(t) = V_0\left(1 - e^{-t/\tau}\right) \quad \text{where } \tau = RC$$

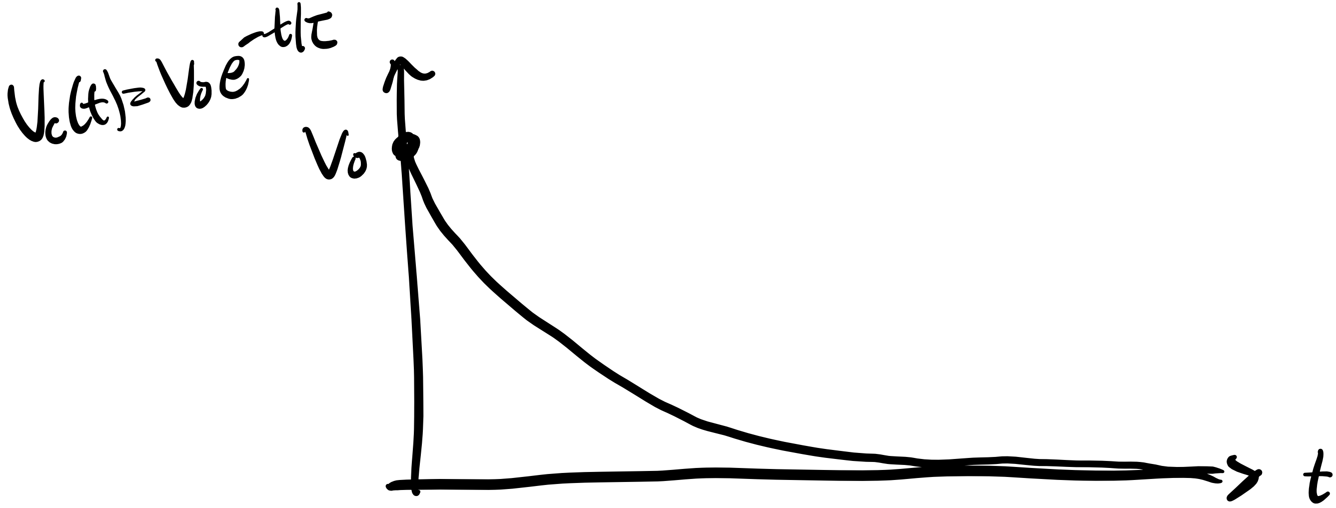


Discharging Capacitor through resistor

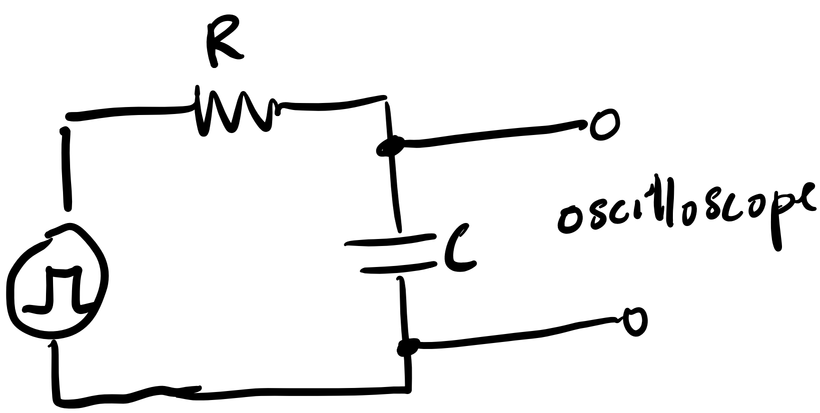


Capacitor initially charged to V_0.
Close switch at time $t=0$.

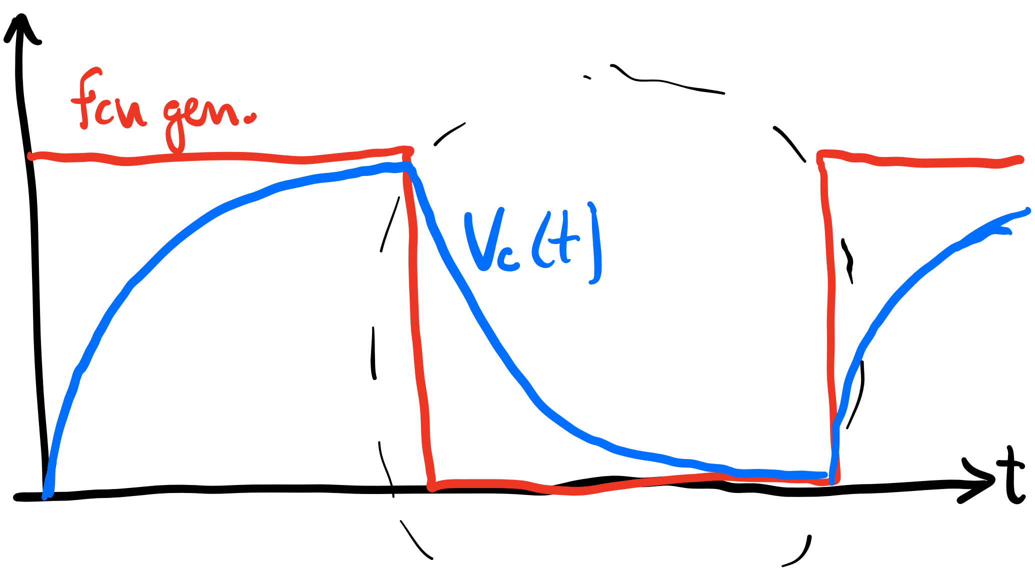
$$V_c(t) = V_0 e^{-t/\tau} \qquad \tau = RC$$



In the Lab, you will construct the following circuit:



fun gen. is set to a low-freq. square wave.



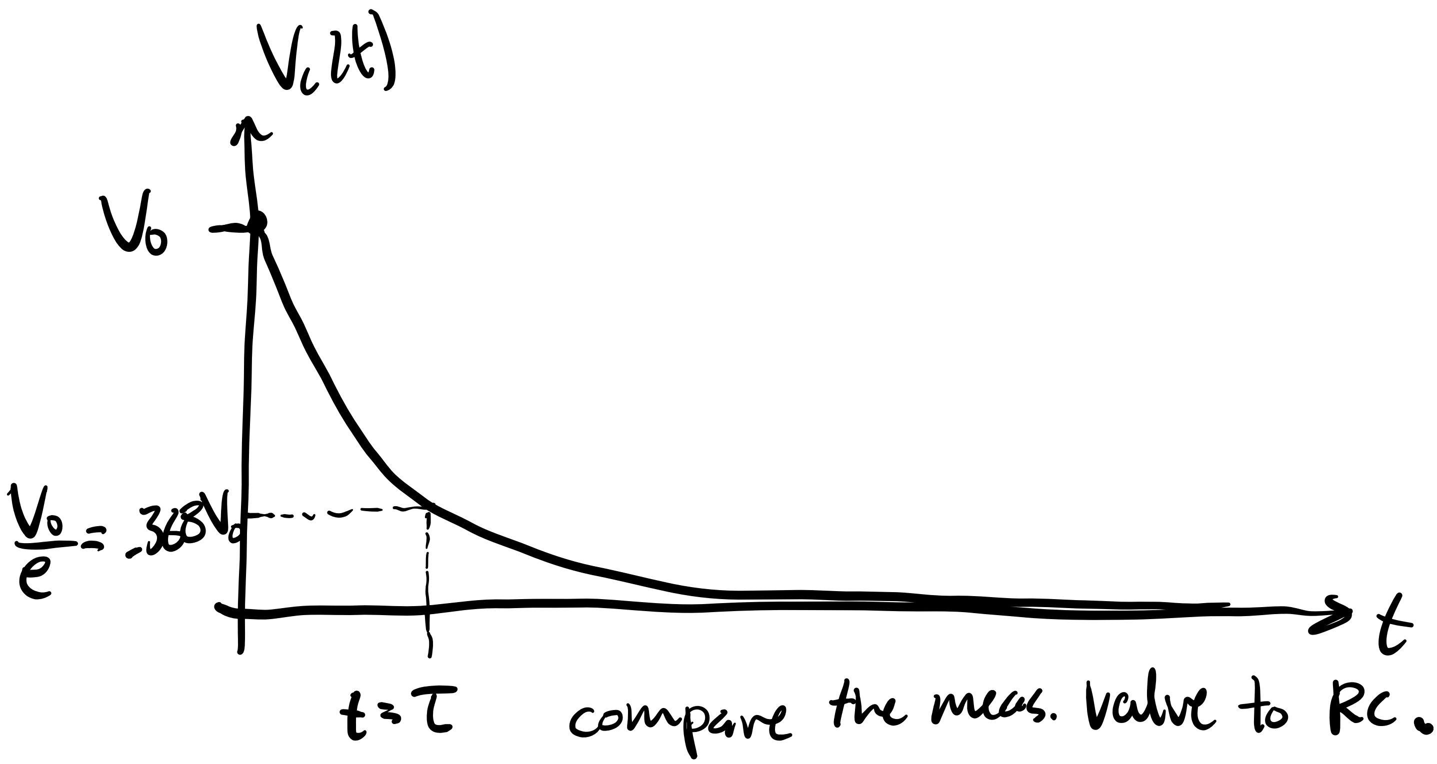
Goal is to make a quick estimate of time const $\tau = RC$.

To estimate τ, find the value of $V_c(t=\tau)$.

$$V_c(t) = V_0 e^{-t/\tau} \quad \text{(discharging)}.$$

$\therefore$ when $t = \tau$

$$V_c(t=\tau) = V_0 e^{-\tau/\tau} = \frac{V_0}{e} = .368\, V_0$$



compare the meas. value to RC.